PREPARED AS PER REDUCED SYLLABUS AND LATEST CBSE SAMPLE PAPER FOR 2021 EXAMINATION

1 Day = 5 Board Questions

100 Day = 500 Board Questions

5. CONTINUITY AND DIFFERENTIABILITY

Day - 1

1. Find the values of a and b such that the function defined by

$$f(x) = \begin{cases} x+2 & if\ \ x \le 2 \\ ax+b & if\ \ 2 < x < 5 \\ 3x-2 & if\ \ x \ge 5 \end{cases}$$ is a continuous function.

2. For what value of k, for which the function f defined below is continuous

$$f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x} & if\ x < \frac{\pi}{2} \\ 3 & if\ x = \frac{\pi}{2} \\ \frac{3\tan 2x}{2x-\pi} & if\ x > \frac{\pi}{2} \end{cases}$$

3. Find the values of a and b such that the following function $f(x)$ is a continuous function $f(x) = \begin{cases} 5 & if\ \ x \le 2 \\ ax+b & if\ \ 2 < x < 10 \\ 21 & if\ \ x \ge 10 \end{cases}$

4. Let $f(x) = \begin{cases} \frac{1-\cos 4x}{x^2} & if\ x < 0 \\ \frac{\sqrt{x}}{(\sqrt{16+\sqrt{x}})-4} & if\ x > 0 \end{cases}$ Determine the value of a so that $f(x)$ is continuous at $x = 0$

5. Let $f(x) = \begin{cases} \frac{1-\sin^3 x}{3\cos^2 x} & if\ x < \frac{\pi}{2} \\ a & if\ x = \frac{\pi}{2} \\ \frac{b(1-\sin x)}{(\pi-2x)^2} & if\ x > \frac{\pi}{2} \end{cases}$ If f(x) is continuous at $x = \frac{\pi}{2}$ find a and b.

Day -2

6. Show that the function f defined as follows, is continuous at $x = 2$, but not differentiable there at $f(x) = \begin{cases} 3x - 2 & if\ 0 < x \le 1 \\ 2x^2 - x & if\ 1 < x \le 2 \\ 5x - 4 & if\ x > 2 \end{cases}$

7. Find the value of k so that the functions f is continuous at $x = \frac{\pi}{2}$

$$f(x) = \begin{cases} \frac{k\ cosx}{\pi - 2x} & if\ x \ne \frac{\pi}{2} \\ 3 & if\ x = \frac{\pi}{2} \end{cases}$$

8. If $f(x) = \begin{cases} \frac{\sin(a+1)x + 2\ sinx}{x} & if\ x < 0 \\ x & if\ x = 0 \\ \frac{\sqrt{1+bx} - 1}{x} & if\ x > 0 \end{cases}$ is continuous at x = 0, then find the values of a and b

9. Show that the function $f(x) = |x - 1| + |x + 1|$ for all $x \in R$, is not differentiable at the points at $x = -1$ and $x = 1$

10. For what value of λ is the function defined by $f(x) = \begin{cases} \lambda(x^2 + 2) & if\ x \le 0 \\ 4x + 6 & if\ x > 0 \end{cases}$ continuous at $x = 0$? Hence check the differentiability of $f(x)$ at $x = 0$

Day -3

11. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\{x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2}\}$

12. If $siny = x\sin(a + y)$, then prove that $\frac{dy}{dx} = \frac{sin^2(a+y)}{sina}$

13. If $\sqrt{1 - x^4} + \sqrt{1 - y^4} = a(x^2 - y^2)$ prove that $\frac{dy}{dx} = \frac{x}{y}\sqrt{\frac{1-y^4}{1-x^4}}$

14. If $e^y(x + 1) = 1$, then show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

15. Prove that $\tan^{-1}\left\{\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right\} = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x \quad if \; -\frac{1}{\sqrt{2}} \le x \le 1$

Day -4

16. If $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log x}{\{\log(xe)\}^2}$

17. If $(\cos x)^y = (\cos y)^x$, find $\frac{dy}{dx}$

18. Differentiate the following w. r. t. x : $x^{\sin x} + (\sin x)^{\cos x}$

19. If $x^m y^n = (x+y)^{m+n}$ prove that (i) $\frac{dy}{dx} = \frac{y}{x}$ (ii) $\frac{d^2y}{dx^2} = 0$

20. Differentiate the following w. r. t. x : $y^x + x^y + x^x = a^b$, $a > 0$, $x > 0$

Day -5

21. Differentiate the following w. r. t. x : $\sin^{-1}\left(\frac{2^{x+1}\,3^x}{1+(36)^x}\right)$

22. Find $\frac{dy}{dx}$, if $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

23. If $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$

24. If $x = a\sin 2t\,(1 + \cos 2t)$ and $y = b\cos 2t\,(1 - \cos 2t)$ find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$

25. If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$, then find the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{4}$

Day – 6(4/01/2021)

26. Find the value of $\frac{dy}{dx}$ at $\theta = \frac{\pi}{4}$ if $x = a\,e^{\theta}(\sin\theta - \cos\theta)$ and $x = a\,e^{\theta}(\sin\theta + \cos\theta)$

27. If $x = 3\cos t - 2\cos^3 t$ and $y = 3\sin t - 2\sin^3 t$ then find $\frac{d^2y}{dx^2}$

28. If $x = a\left(\cos t + \log\tan\frac{t}{2}\right)$, $y = a\sin t$ evaluate $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$

29. Differentiate $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ with respect to $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ Solution Click Here

30. Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $sin^{-1}\left(\frac{2x}{1+x^2}\right)$ when $x \neq 0$ Solution Click Here

Day – 7 (5/01/2021)

31. If $y = (\sec^{-1} x)^2$, then show that $x^2(x^2-1)\frac{d^2y}{dx^2} + (2x^3 - x)\frac{dy}{dx} = 2$ Solution Click Here

32. If $y = e^{m\cos^{-1}x}$, $-1 \le x \le 1$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$ Solution Click Here

33. If $y = \left\{x + \sqrt{x^2+1}\right\}^m$ show that $(x^2+1)y_2 + xy_1 - m^2y = 0$ Solution Click Here

34. If $y = 2\cos(logx) + 3\sin(logx)$, prove that $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ Solution Click Here

35. If $(x-a)^2 + (y-b)^2 = c^2$, for some $c > 0$, prove that $\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\frac{d^2y}{dx^2}}$ is a constant independent of a and b Solution Click Here

Day – 8 (6/01/2021)

36. Find the intervals in which the function f given by $f(x) = sin\,x + cos\,x$, $0 \le x \le 2\pi$ is strictly increasing or strictly decreasing Solution Click Here

37. Find the intervals in which the function f given by $f(x) = \frac{3}{10}x^4 - \frac{4}{5}x^3 - 3x^2 + \frac{36}{5}x + 11$ is (a) strictly increasing (b) strictly decreasing Solution Click Here

38. Find the intervals in which the function f given by $f(x) = sin3x - cos\,3x$, $0 \le x \le \pi$ is strictly increasing or strictly decreasing Solution Click Here

39. Find the intervals in which the function f given by $f(x) = \frac{4\,sinx - 2x - x\,cosx}{2 + cosx}$, $0 \le x \le 2\pi$ are (a) strictly increasing (b) strictly decreasing. Solution Click Here

40. On which of the following intervals is the function f given by $f(x) = x^{100} + sinx - 1$ Strictly decreasing? Solution Click Here

(a) (0, 1) (b) $\left(\frac{\pi}{2}, \pi\right)$ (c) $\left(0, \frac{\pi}{2}\right)$ (d) None of these

Next Day – 9 (7/01/2021)

41. Find the equation of tangent to the curve $y = x^3 + 2x + 6$ which is perpendicular to the line $x + 14y + 4 = 0$

42. For the curve $x = 2cos\theta - cos2\theta$ and $y = 2sin\theta - sin2\theta$, find the equation of tangent at $\theta = \frac{\pi}{4}$

43. Show that the equation of normal at any point on the curve $x = 3cos\,\theta - cos^3\theta,\ y = 3sin\theta - sin^3\theta$ is $4\,(y\,cos^3\theta - x\,sin^3\theta) = 3\,sin\,4\theta$

44. Find the equation of tangents to the curve $y = \cos(x + y),\ -2\pi \le x \le 0$ that are parallel to the line $x + 2y = 0$

45. For the curve $y = 4x^3 - 2x^5$, find all the points on the curve at which the tangent passes through the origin

Next Day – 10 (8/01/2021)

46. Show that the normal at any point θ to the curve $x = a\cos\theta + a\,\theta\sin\theta$, $y = a\sin\theta - a\,\theta\cos\theta$ is at a constant distance from the origin.

47. Find the equations of the tangent and normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$

48. Prove that the curves $x = y^2$ and $xy = k$ cut at right angles if $8k^2 =$

49. Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is (a) parallel to the line $2x - y + 9 = 0$ (b) perpendicular to the line $5y - 15x = 13$

50. Find the values of p for which the curves $x^2 = 9p(9 - y)$ and $x^2 = p(y + 1)$ cut each other at right angles

Next Day – 11 (9/01/2021)

51. Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius R is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume

52. If the sum of the length of the hypotenuse and a side of a right angle triangle is given, show that the area of the triangle is maximum when the angle between them is $\frac{\pi}{3}$

53. AB is a diameter of a circle and C is any point on the circle. Show that the area of ΔABC is maximum, when it is isosceles

54. A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening

55. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere

(OR)

Show that the altitude of the right circular cone of maximum volume that can be inscribed in a sphere of radius r is $\frac{4r}{3}$

Next Day – 12 (10/01/2021)

56. If length of three sides of a trapezium other than base are equal to 10cm, then find the area of the trapezium when it is maximum

57. Prove that the radius of the right circular cylinder of greatest curved surface area which can be inscribed in a given cone is half of that of the cone

58. Show that the right circular cone of least curved surface and given volume has an altitude equal to $\sqrt{2}$ time the radius of the base

59. Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $tan^{-1}\sqrt{2}$ (OR)

Show that the semi-vertical angle of the cone of the maximum volume and of given slant height is $\cos^{-1}\frac{1}{\sqrt{3}}$

60. Show that height of the cylinder of greatest volume which can be inscribed in a right circular cone of height h and semi-vertical angle α, is one –third that of the cone. Hence find the greatest volume of the cylinder

Day – 13 (11/01/2021)

61. Using integration, find the area of the region bounded by the following curves, after aking a rough sketch $y = 1 + |x + 1|,\ x = -3,\ x = 3,\ y = 0$

62. Find the area of the region $\{(x, y) :\ x^2 + y^2 \le 1 \le x + y\}$ by using integration

63. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the line $\frac{x}{3} + \frac{y}{2} = 1$ (OR)

64. Using integration, find the area of the region $\left\{(x, y) : \frac{x^2}{9} + \frac{y^2}{4} \le 1 \le \frac{x}{3} + \frac{y}{2}\right\}$

65. Using integration, find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$

Day – 14 (12/01/2021)

66. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are $A(\ 4,\ 1\ ),\ B\ (6,\ 6)\ and\ C\ (8,\ 4)$.

67. Using the method of integration find the area of the region bounded by lines: $2x + y = 4,\ 3x - 2y = 6$ and $x - 3y + 5 = 0$

68. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are $A(\ -2,\ 1\ ),\ B\ (0,\ 4)\ and\ C\ (2,\ 3)$

69. Using the method of integration find the area of the region bounded by lines: $3x - y - 3 = 0,\ 2x + y - 12 = 0,\ -2y - 1 = 0$

70. Using the method of integration find the area of the triangle ABC, coordinates of whose vertices are $A(\ 1, -2\ ),\ B\ (3,\ 5)\ and\ C\ (5,\ 2)$

Day – 15 (13/01/2021)

71. Find the area of the region bounded by the curve $y^2 = 4x$ and the line $y = x$

72. Find the area of the region in the first quadrant enclosed by the x-axis, the line $y = x$, and the circle $x^2 + y^2 = 32$

73. Using integration, find the area enclosed by the parabola $4y = 3x^2$ and the line $2y = 3x + 12$

74. Find the area of the smaller region bounded by the ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ and the straight line $3x+4y=12$

75. Sketch the graph $y=|x+1|$ and using integration find the area below $y=|x+1|$, above x- axis and between $x=-4$ to $x=2$

Day – 16 (14/01/2021)

76. Find the particular solution of the differential equation $(1+e^{2x})\,dy+(1+y^2)\,e^x\,dx=0$, given that $y=1$ when $x=0$.

77. Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{x\,(2\,logx+1)}{siny+y\,cosy}$ given that $y=\frac{\pi}{2}$ when $x=1$

78. Find the particular solution of the differential equation $(1-y^2)\,(1+logx)\,dx+2xy\,dy=0$ given that $y=0$ when $x=1$

79. Find the general solution of the differential equation $y-x\frac{dy}{dx}=a\left(y^2+\frac{dy}{dx}\right)$

80. Solve the following differential equation $(1+y^2)\,(1+logx)\,dx+x\,dy=0$

Day – 17 (15/01/2021)

81. Solve the following differential equation $\left\{x\,cos\left(\frac{y}{x}\right)+y\,sin\left(\frac{y}{x}\right)\right\}y-\left\{y\,sin\left(\frac{y}{x}\right)-x\,cos\left(\frac{y}{x}\right)\right\}x\frac{dy}{dx}=0$

82. Solve $\left[x\,sin^2\left(\frac{y}{x}\right)-y\right]dx+x\,dy=0$; when $y=\frac{\pi}{4}$ and x =1

83. Prove that $x^2-y^2=c\,(x^2+y^2)$ is the general solution of the differential equation $(x^3-3xy^2)\,dx=(y^3-3x^2y)\,dy$ where C is a parameter

84. Solve the following differential equation $x\,cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\,cos\left(\frac{y}{x}\right)+x$, $x\neq 0$

85. Solve $\left[x\,sin^2\left(\frac{y}{x}\right)-y\right]dx+x\,dy=0$; when $y=\frac{\pi}{4}$ and x =1

Day – 18 (16/01/2021)

86. Show that the differential equation $2y\, e^{\frac{x}{y}}\, dx + \left(y - 2\, x\, e^{\frac{x}{y}}\right)\, dy = 0$ homogeneous and find its particular solution, given that, $x = 0$ when $y = 1$.

87. Find a particular solution of the following differential equation $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x\,,\ x \neq 0$ given that $y = \frac{\pi}{4}$ and x =1

88. Find a particular solution of the following differential equation $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ given that $y = 1$ when $x = 0$

89. Show that the differential equation $x\frac{dy}{dx}\sin\left(\frac{y}{x}\right) + x - y\sin\left(\frac{y}{x}\right) = 0$ homogeneous and find its particular solution, given that, $x = 1$ when $y = \frac{\pi}{2}$

90. Find the general solution of the differential equation $2xy + y^2 - 2x^2\frac{dy}{dx} = 0;$ $y = 2$ when $x = 1$

Day – 19 (17/01/2021)

91. Solve $(1 + x^2)\frac{dy}{dx} + 2xy = \frac{1}{1+x^2}$ $y = 2\ when\ x = 1$

92. Solve $\frac{dy}{dx} + y\cot x = 2x + x^2\cot x,\ x \neq 0,$ given that $y = 0,$ when $x = \frac{\pi}{2}$

93. Solve the following differential equation $x\frac{dy}{dx} + y = x\cos x + \sin x$, given $y\left(\frac{\pi}{2}\right) = 1$

94. Find the general solution of the differential equation $y\, dx - (x + y^2)\, dy = 0$

95. Solve the following differential equation $\cos^2 x\frac{dy}{dx} + y = \tan x$ $\left(0 \leq x \leq \frac{\pi}{2}\right)$

Day – 20 (18/01/2021)

96. Find a particular solution of the differential equation $(\tan^{-1}y - x)\, dy = (1 + y^2)dx,$ given that when $x = 1,\ y = 0$

97. Find a particular solution of the differential equation $\frac{dy}{dx} + x\cot y = 2y + y^2\cot y\,,\ (y \neq 0)$, given that $x = 0$ when $y = \frac{\pi}{2}$

98. Solve the following differential equation $x\frac{dy}{dx} + y - x + xy\,cotx = 0 \quad (x \neq 0)$ given that $x = \frac{\pi}{2}$, y = 0

99. Solve the following differential equation $x\frac{dy}{dx} + y = x\,cosx + sinx$ given that $y = 1$ when $x = \frac{\pi}{2}$

100. Find a particular solution of the differential equation $(x - y)\frac{dy}{dx} = (x + 2y)$ given that $y = 0$ when $x = 1$

Day – 21 (19/01/2021)

101. Find the position vector of a point R which divides the line joining the two points P and Q with position vectors $\overrightarrow{OP} = 2\vec{a} + \vec{b}$ and $\overrightarrow{OQ} = \vec{a} - 2\vec{b}$, respectively, externally in the ratio $1:2$. Also, show that P is the mid point of the line segment RQ

102. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$, and $\vec{b} = \vec{i} - 2\vec{j} + \vec{k}$

103. Using vectors show that the points $A(-2,3,5), B(7,0,-1), (-3,-2,-5)$ *and* $D(3,4,7)$ are such that AB and CD intersect at the point $P(1,2,3)$

104. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three vectors of magnitudes 3, 4 and 5 respectively such that each one is perpendicular to the sum of the other two vectors, prove that $|\vec{a} + \vec{b} + \vec{c}| = 5\sqrt{2}$

(OR)

105. Let $\vec{a}$, $\vec{b}$ *and* $\vec{c}$ be three vectors such that $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ and each one of them being perpendicular to the sum of the other two, find $|\vec{a} + \vec{b} + \vec{c}|$

Day – 22 (20/01/2021)

106. If two vector $\vec{a}$, *and* $\vec{b}$ are such that $|\vec{a}| = 2$, $|\vec{b}| = 1$ and $\vec{a}\,.\vec{b}$ then find the value of $(3\vec{a} - 5\vec{b}) \cdot (2\vec{a} + 7\vec{b})$

107. Let $\vec{a}$, $\vec{b}$ *and* $\vec{c}$ be three vectors such that $|\vec{a}| = 5$, $|\vec{b}| = 12$, and $|\vec{c}| = 13$, and $\vec{a} + \vec{b} + \vec{c} = 0$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$

108. If $\vec{a} = \vec{i} - \vec{j} + 7\vec{k}$ and $\vec{b} = 5\vec{i} - \vec{j} + \lambda\vec{k}$, then find the value of λ, so that $\vec{a} + \vec{b}$ *and* $\vec{a} - \vec{b}$ are perpendicular vectors

109. If $\vec{a} = 3\vec{i} - \vec{j}$ and $\vec{b} = 2\vec{i} + \vec{j} - 3\vec{k}$ then express $\vec{b}$ in the form $\vec{b} = \vec{b}_1 + \vec{b}_2$, where $\vec{b}_1$ is parallel to $\vec{a}$ and $\vec{b}_2$ is perpendicular to $\vec{a}$

110. The scalar product of the vector $\vec{a} = \vec{i} + \vec{j} + \vec{k}$ with a unit vector along the sum of vectors $\vec{b} = 2\vec{i} + 4\vec{j} - 5\vec{k}$ and $\vec{c} = \lambda\vec{i} + 2\vec{j} + 3\vec{k}$ is equal to one. Find the value of λ and hence find the unit vector along $\vec{b} + \vec{c}$

Day – 23 (21/01/2021)

111. Let $\vec{a} = \vec{i} + 4\vec{j} + 2\vec{k}$, $\vec{b} = 3\vec{i} - 2\vec{j} + 7\vec{k}$ and $\vec{c} = 2\vec{i} - \vec{j} + 4\vec{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 18$

(OR)

Let $\vec{a} = \vec{i} + 4\vec{j} + 2\vec{k}$, $\vec{b} = 3\vec{i} - 2\vec{j} + 7\vec{k}$ and $\vec{c} = 2\vec{i} - \vec{j} + 4\vec{k}$. Find a vector $\vec{p}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{p} = 18$

112. Find a unit vector perpendicular to each of the vector $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$, where $\vec{a} = 3\vec{i} + 2\vec{j} + 2\vec{k}$, and $\vec{b} = \vec{i} + 2\vec{j} - 2\vec{k}$

113. Let $\vec{a} = 4\vec{i} + 5\vec{j} - \vec{k}$, $\vec{b} = \vec{i} - 4\vec{j} + 5\vec{k}$ and $\vec{c} = 3\vec{i} + \vec{j} - \vec{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 21$

114. Let $\vec{a} = 4\vec{i} + 5\vec{j} - \vec{k}$, $\vec{b} = \vec{i} - 4\vec{j} + 5\vec{k}$ and $\vec{c} = 3\vec{i} + \vec{j} - \vec{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and $\vec{c} \cdot \vec{d} = 21$

(OR)

Find the vector $\vec{p}$ which is perpendicular to both $\vec{\alpha} = 4\vec{i} + 5\vec{j} - \vec{k}$ and $\vec{\beta} = \vec{i} - 4\vec{j} + 5\vec{k}$ and $\vec{p} \cdot \vec{q} = 21$, where $\vec{q} = 3\vec{i} + \vec{j} - \vec{k}$

115. Show that the points A, B, C with position vectors, $2\vec{i} - \vec{j} + \vec{k},\ \ 3\vec{i} - 4\vec{j} - 4\vec{k}$ respectively are the vertices of a right angled triangle. Hence find the area of the triangle

Day – 24 (22/01/2021)

116. Find the position vector of a point A in space such that $\overrightarrow{OA}$ is inclined at 60° to OX and at 45° to OY and $|\overrightarrow{OA}| = 10$ units.

117. A line makes angles $\alpha, \beta, \gamma, \delta$ with the diagonals of a cube, prove that $cos^2\alpha + cos^2\beta + cos^2\gamma + cos^2\delta = \frac{4}{3}$

118. Find the vector equation of the line passing through the point (1, 2, – 4) and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

119. Find the vector equation of the line passing through the point $P(2, -1, 3)$ and perpendicular to the two lines: $\vec{r} = (\vec{i} + \vec{j} - \vec{k}) + \lambda(2\vec{i} - 2\vec{j} + \vec{k})$ and $\vec{r} = (2\vec{i} - \vec{j} - 3\vec{k}) + \mu(\vec{i} + 2\vec{j} + 2\vec{k})$

120. Find the vector equation of the line passing through the point (1, 2, – 4) and perpendicular to the two lines: $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

Day – 25 (23/01/2021)

121. Find the shortest distance between the two lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-3}{3} = \frac{y-3}{4} = \frac{z-5}{5}$

122. Find the shortest distance between the two lines $\vec{r} = (1-t)\vec{i} + (t-2)\vec{j} + (3-2t)\vec{k}$ and $\vec{r} = (s+1)\vec{i} + (2s-1)\vec{j} - (2s+1)\vec{k}$

123. Show that the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-1}{2} = z$ intersect. Find their point of intersection.

124. Show that the lines $\vec{r} = (\vec{i} + \vec{j} - \vec{k}) + \lambda(3\vec{i} - \vec{j})$ and $\vec{r} = 4\vec{i} - \vec{k} + \mu(2\vec{i} + 3\vec{j})$ intersect. Find the point of intersection.

125. Prove that the lines through $A\ (0,-1,-1)$ and $B\ (4,\ 5,\ 1)$ intersects the line through C$(3,\ 9,4)$ and $D\ (-4,\ 4,\ 4)$. Also, find the their point intersection.

Day – 26 (24/01/2021)

126. Find the point on the line (x + 2)/3= (y +1)/2 = (z - 3)/2 at a distance of 5 units from the oint P(1,3,3)

127. Find the perpendicular distance from the point (1,0 ,0) to the line (x -1)/2= (y +1)/(-3)= (z + 10)/8. Also, find the coordinates of the foot of the perpendicular and the equation of the perpendicular.

128. Find the equation of perpendicular from the point (3,-1 ,11) to the line (x)/2= (y - 1)/3= (z - 3)/4 . Also, find the coordinates of the foot of the perpendicular and the length of the perpendicular.

129. Find the equation of a line passing through the points A(0,6,-9) and B(-3,-6,3). If D is the foot of perpendicular drawn from a point C(7,4,-1) on the line AB, then find the coordinates of the point D and the equation of line CD.

130. Find the coordinates of the foot of the perpendicular and the length of the perpendicular drawn from the point P(5,4,2) to the line $\vec{r} = (-\vec{i} + 3\vec{j} + \vec{k}) + \lambda\,(2\vec{i} + 3\,\vec{j} - \vec{k})$. Find also, the image of P in the line

Day – 27 (25/01/2021)

131. Find the equation of the plane passing through the point $(\ 1,\ 1,-1)$ and perpendicular to each of the planes $x\ +\ 2y\ +\ 3z - 7 =\ 0$ and $2x -\ 3y\ +\ 4\,z\ =\ 0.$

132. Find the cartesian and vector equations of the plane passing through the points $A(2,5,-\ 3),\ B(-2,-\ 3,5)\ and\ C(5,\ 3,-3)$

133. Find the equation of the plane determined by the point $A(\ 3,-1,2),\ B(5,2,4)$ and $C(-1,-1,\ 6)$ and hence find the distance between the plane and the point $P(6,\ 5,\ 9)$

134. Find the distance between the line $\frac{x-5}{3} = \frac{y-4}{3} = \frac{z-8}{1}$ and the plane determined by the points $A(2,2,1),\ B(4,\ 1,3)$ and $C(-2,-2,5)$

135. Find the vector and Cartesian equation of the plane passing through the point $(-1, 3, 2)$ and perpendicular to each of the planes $x + 2y + 3z = 5$ and $3x + 3y + z = 0$

Day – 28 (26/01/2021)

136. Find the vector equation of the plane passing through the intersection of the planes $x + 3y + 6 = 0$ and $3x - y - 4z = 0$ and whose perpendicular distance from the origin is unity.

137. Find the equation of the plane which contains the line of intersection of the planes $\vec{r}.(\vec{i} + 2\vec{j} + 3\vec{k}) - 4 = 0$ and $\vec{r}.(2\vec{i} + \vec{j} - \vec{k}) + 5 = 0$ which is perpendicular to the plane $\vec{r}.(5\vec{i} + 3\vec{j} - 6\vec{k}) + 8 = 0$

138. Find the equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$. Also find distance of the plane obtained above, from the point $A(1, 3, 6)$

139. Find the equation of the plane through the line of intersection of $\vec{r}.(2\vec{i} - 3\vec{j} + 4\vec{k}) = 1$ and $\vec{r}.(\vec{i} - \vec{j}) + 4 = 0$ and perpendicular to the plane $\vec{r}.(2\vec{i} - \vec{j} + \vec{k}) + 8 = 0$. Hence find the where the plane thus obtained contains the line $x - 1 = 2y - 4 = 3z - 12$

140. A variable plane which remains at a constant distance 3p from the origin cut the coordinate axes at A, B, C. Show that the locus of the centroid of triangle ABC is $x^{-2} + y^{-2} + z^{-2} = p^{-2}$

Day – 29 (27/01/2021)

141. Solve the following LPP graphically Minimise $Z = 5x + 10y$ Subject to Constraints $x + 2y \leq 120; x + y \geq 60, x - 2y \geq 0$, and $x, y \geq 0$
[Ans : Minimum value = 300 at x = 60, y = 0]

142. Solve the following L.P.P. graphically : Maximizes $Z = 20x + 10y$ Subject to following constraints $x + 2y \leq 28, 3x + y \leq 24; x \geq 2; x, y \geq 0$

[Ans : Maximum value of Z = 200 at $x = 4,\ y = 12$]

143. Find graphically, the maximum value of Z = $2x + 5y$, subject to constraints given below: $2x + 4y \leq 8,\ 3x + y \leq 6,\ x + y \leq 4,\ x \geq 0,\ y \geq 0$

[Ans : Maximum value of Z = 9.2 at $x = \frac{8}{5},\ y = \frac{6}{5}$]

144. Solve the following linear programming problem graphically: Maximize $Z = 7x + 10y$ Under the following constraints $4x + 6y \leq 240,\ 6x + 3y \leq 240,\ x \geq 10,\ x \geq 0,\ y \geq 0$

145. Solve the following Linear Programming problem graphically : Minimize : $Z = 3x + 9y$ When : $x + y \leq 50, 2x + y \leq 80,\ x \geq 20,\ x > 0,\ y > 0$

[Ans : Maximum value of Z = 4950 at $x = 30,\ y = 20$]

Day – 30 (28/01/2021)

146. One kind of cake requires 200 g of flour and 25 g 3 hours o of fat and another kind of cake requires 100 g of produce flour and 50 g of fat. Find the maximum number 17.50 pe of cakes which can be made from 5 kg of flour bolts. E and 1 kg of fat, assuming that there is no shortage proud of other ingredients used in making the cakes. if he Formulate the above as a linear programming problem and solve it graphically. **[Ans : Maximum number of cakes is 30 at $x = 20,\ y = 10$]**

147. If a young man drives his scooter at a speed of 25 km/hour, he has to spend 2 per km on petrol. If he drives the scooter at a speed of 40 km/hour, it produces more air pollution and increases his expenditure on petrol to 5 per km. He has a maximum of 100 to spend on petrol and travel a maximum distance in one hour time with less pollution. Express this problem as an LPP and solve it graphically. **[Ans : Maximum distance 30, at $x = \frac{50}{3}$ km, $y = \frac{40}{3}$ km]**

148. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1. of gold while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of `40 and that of type B `50, find the number of units of each type that the company should produce to maximize profit. Formulate the above LPP and solve it graphically and also find the maximum profit. **[Maximum profit `230 at x = 2, y = 3]**

149. A manufacturer has employed 5 skilled men and 10 semi-skilled men and makes two models A and B of an article. The making of one item of model A requires 2 h work by a skilled man and 2 h work by a semi-skilled man. One item of model B requires 1 h by a skilled

man and 3 h by a semi-skilled man. No man is expected to work more than 8 h per day. The manufacturer profit on an item of model A is `15 and on an items of model B is `10. How many of items of each models should be made per day in order to maximize daily profit? Formulate the above LPP and solve it graphically and find the maximum profit. **[Maximum profit `350 at x = 10, y = 20]**

150. A company produces two types of goods, A and B, that require gold and silver. Each unit of type A requires 3 g of silver and 1. of gold while that of type B requires 1 g of silver and 2 g of gold. The company can use at the most 9 g of silver and 8 g of gold. If each unit of type A brings a profit of `40 and that of type B `50, find the number of units of each type that the company should produce to maximize profit. Formulate the above LPP and solve it graphically and also find the maximum profit. **[Maximum profit `230 at x = 2, y = 3]**

Day – 31 (29/01/2021)

151. Assume that each born child is equally likely to be a boy or a girl. If a family has two children. what is the conditional probability that both are girls ? Given that (i) the youngest is a girl (ii) at least one is a girl

152. A die is thrown three times. Events A and B are defined as below:
A: 5 on the first and 6 on the second throw, B: 3 or 4 on the third throw. Find the probability of B, given that A has already occurred.

153. The probabilities of two students A and 3 coming to the school in time $\frac{3}{7}$ and $\frac{5}{7}$ respectively. Assuming that the events 'A coming in time' and 'B coming in time' are independent, find the probability of only one of them coming to the school in time. Write at least one advantage of coming to school in time

154. P speaks truth in 70% of the cases and Q in 80% of the cases. In what per cent of cases are they likely to agree in stating the same fact? Do you think, when they agre, means both are speaking truth?

155. A speaks truth in 60% of the cases, while B in 90% of the cases. In what per cent of cases are they likely to contradict each other in stating the same fact? In the cases of contradiction do you think the statement of B will carry more weight as he speaks number of cases than A?

PREPARED AS PER REDUCED SYLLABUS AND LATEST CBSE SAMPLE PAPER FOR 2021 EXAMINATION

Day – 32 (30/01/2021)

156. Probability of solving specific problem independently by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability that
   (i) the problem is solved (or) at least one of them solved
   (ii) exactly one of them solves the problem.

157. A and B throw a die alternatively till one of them gets a '6' and wins the game. Find their respective probabilities of winning, if A starts first.

158. A and B throw a pair of dice alternately. A wins the game if he gets a total of 6 and B wins if she gets a total of 7. It A starts the game, find the probability of winning the game by A in third throw of the pair of dice.

159. Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that
   (i) both balls are red. (ii) first ball is black and second is red. (iii) one of them is black and other is red

160. **Consider the experiment of throwing a die, if a multiple of 3 comes up, throw the die again and if any other number comes, toss a coin. Find the conditional probability of the event**

Day – 33 (31/01/2021)

161. There are three coins. One is a two-headed coin (having head on both faces), another is a biased coin that comes up heads 75% of the times and third is also a biased coin that comes up tails 40% of the times. One of the three coins is chosen at random and tossed, and it shows heads. What is the probability that it was the two-headed coin?

162. A card from a pack of 52 playing cards is lost. From the remaining cards of the pack three cards are drawn at random (without replacement) and are found to be all spades. Find the probability of the lost card being a spade.

163. 40% students of a college reside in hostel and the remaining reside outside. At the end of the year, 50% of the hostellers got A grade while from outside students, only 30% got A grade in the examination. At the end of the year, a student of the college was chosen at random and was found to have gotten A grade. What is the probability that the sclected student was a hosteller?

164. An urn contains 3 red and 5 black balls. A ball is drawn at random, its colour is noted and returned to the urn. Moreover, 2 additional balls of the colour noted down, are put in the urn and then two balls are drawn at random (without replacement) from the urn, Find the probability that both the balls drawn are of red colour.

165. Bag A contains 3 red and 5 black balls, while bag B contains 4 red and 4 black balls. Two balls are transferred at random from bag A to bag B and then a ball is drawn from bag B at random. If the ball drawn from bag B is found to be red, find the probability that two red balls were transferred from bag A to B.